Houston ISD ATeaMS
Second Six-Weeks HAPG Study

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Sana Brennan
Region 4 Education Service Center
sbrennan@esc4.net
713.744.4401

region 4®
Educated Solutions

Handouts will be available online until 09.29.09

http://www.theansweris4.net
-Click on Departments
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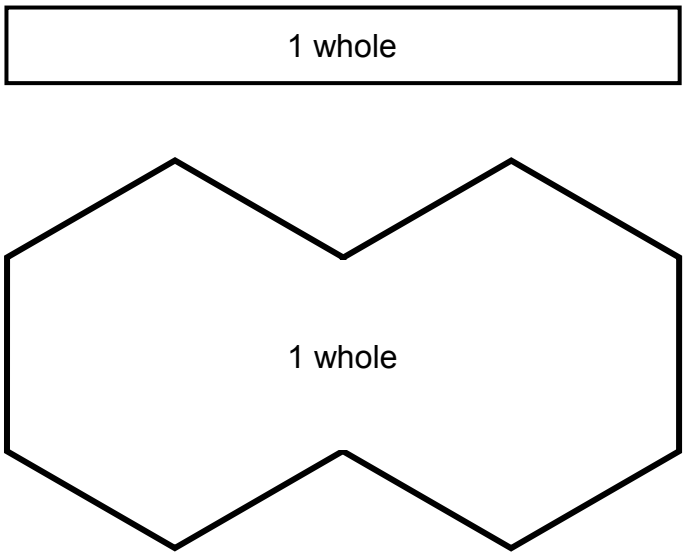
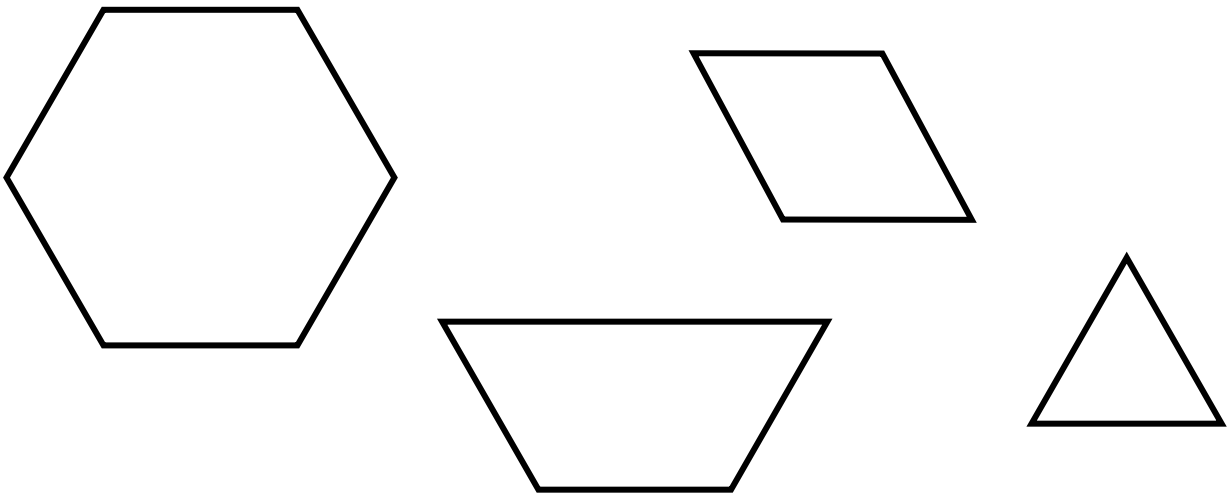
Pattern Blocks

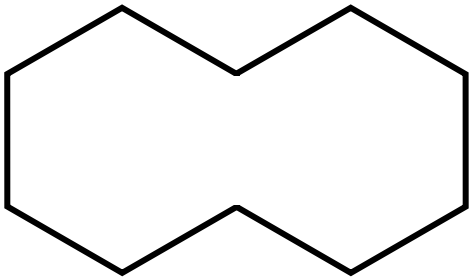
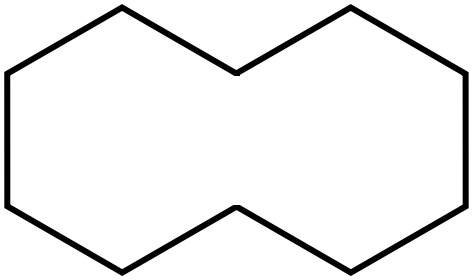
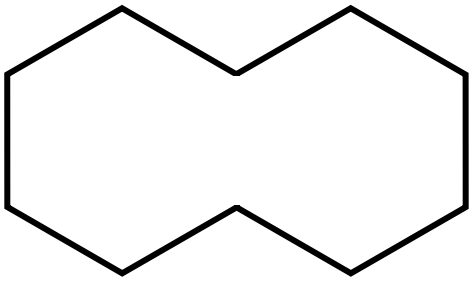
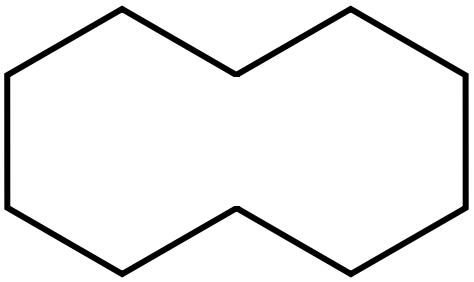


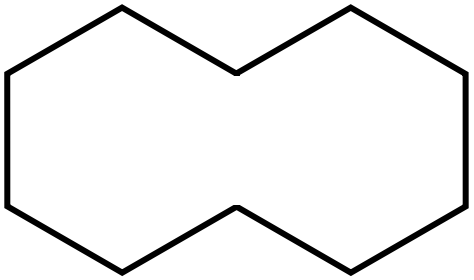
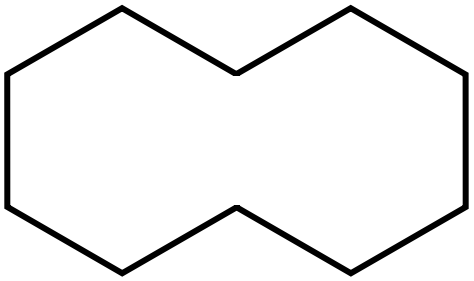
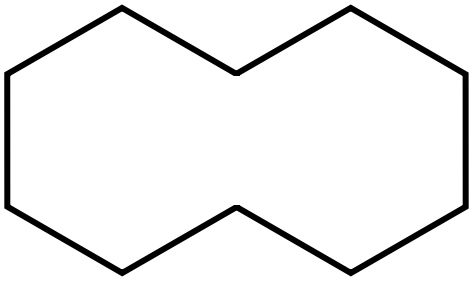
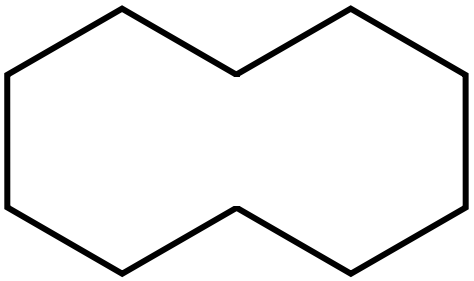
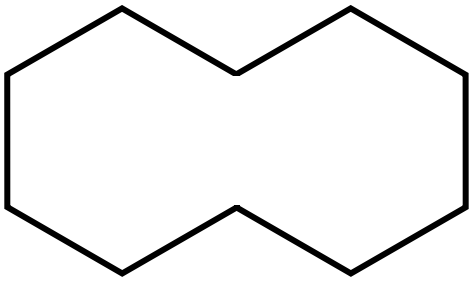
If 2 hexagons represent 1 whole, write the fractional value of each piece inside each shape.



Adding Fractions with Pattern Blocks

Use your pattern blocks to help you determine the solution of each addition problem below. For each problem, the figure represents 1 whole.

	Problem	Model with Pattern Blocks	Number Sentence
1.	$\frac{1}{4} + \frac{3}{4}$		
2.	$\frac{2}{6} + \frac{3}{6}$		
3.	$\frac{1}{12} + \frac{5}{12}$		
4.	$\frac{1}{2} + \frac{1}{4}$		

	Problem	Model with Pattern Blocks	Number Sentence
5.	$\frac{1}{6}+\frac{1}{12}$		
6.	$\frac{1}{4}+\frac{1}{6}$		
7.	$\frac{1}{2}+\frac{1}{12}$		
8.	$\frac{1}{4}+\frac{2}{6}$		
9.	$\frac{3}{4}+\frac{1}{12}$		

Missing Parts

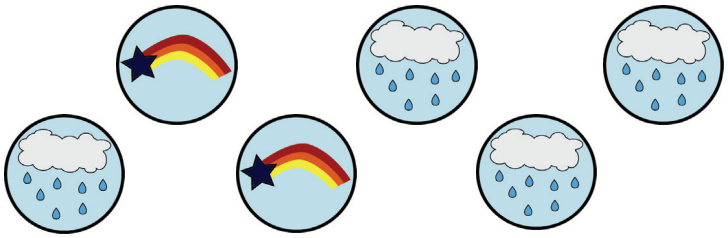
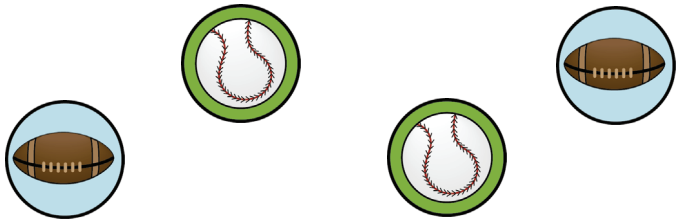
Use Cuisenaire® Rods to find the missing number and record your observations.

Equivalent Fractions	Equivalent Fraction Models	Observations
$2\frac{1}{3} = \square\frac{4}{3}$		
$\square\frac{1}{2} = 3\frac{3}{2}$		
$2\frac{2}{5} = 1\frac{\square}{5}$		
$3\frac{1}{\square} = 2\frac{4}{\square}$		

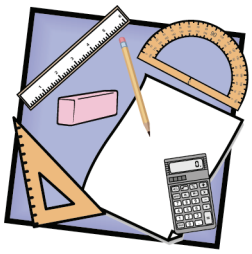
Student Name: _____ Date: _____

Susie's Sticker Collection

Susie wants to organize each of her 3 sticker collections below. She wants to pair the stickers in each collection so that she has 1 of each type of sticker per pair. She will then discard any extra stickers. Circle the pairs of stickers Susie will keep and cross off the discarded stickers in each collection below.

	Sticker Collections
1	
2	
3	

Communicating about Mathematics
Explain the strategy you used to determine which stickers would be discarded.



Scuba Diving

When Janie went scuba diving, she descended 3 feet every minute. What integer represents her depth after one-half hour? Justify your answer.

Backyard Spa

The thermometer in Belinda's backyard spa read 66°F before the heater was turned on. After the heater was turned on, the temperature rose 5°F every 5 minutes. If her thermometer now reads 101°F, how many minutes have passed? Justify your answer.

Complete the table

- Write a description of the problem.
- Draw a picture to model the problem using two-color counters.
- Draw a picture to model the problem using a number line.
- Determine the value of the collection.
- Write 2 different division number sentences related to the multiplication problem.

Problem	Description	Picture Two-Color Counters	Picture Number Line	Solution
2 x 3				
Write 2 different division number sentences related to the fact family of 2 x 3 = _____				
4 x 2				
Write 2 different division number sentences related to the fact family of 4 x 2 = _____				

When a positive integer is **multiplied** by a positive integer the result is _____.

When a positive integer is **divided** by a positive integer the result is_____.

Problem	Description	Picture Two-Color Counters	Picture Number Line	Solution
2 x (−3)				
Write 2 different division number sentences related to the fact family of 2 x (−3) = _____				
4 x (−2)				
Write 2 different division number sentences related to the fact family of 4 x (−2) = _____				

When a positive integer is **multiplied** by a negative integer the result is _____.

When a negative integer is **divided** by a positive integer the result is_____.

When a negative integer is **divided** by a negative integer the result is_____.

Problem	Description	Picture Two-Color Counters	Picture Number Line	Solution
−2 x 3				
Write 2 different division number sentences related to the fact family of −2 x 3 = _____				
−4 x 2				
Write 2 different division number sentences with the fact family of −4 x 2 = _____				

When a negative integer is **multiplied** by a positive integer the result is _____.

When a negative integer is **divided** by a positive integer the result is _____.

Problem	Description	Picture Two-Color Counters	Picture Number Line	Solution
−2 x (−3)				
Write 2 different division number sentences related to the fact family of −2 x (−3) = _____				
−4 x (−2)				
Write 2 different division number sentences related to the fact family of −4 x (−2) = _____				

When a negative integer is **multiplied** by a negative integer the result is _____.

When a positive integer is **divided** by a negative integer the result is _____.

Generalize:

When 2 integers with the **same sign** are multiplied the result will be a _____integer.

When 2 integers with the **same sign** are divided the result will be a _____ integer.

When 2 integers with **different signs** are multiplied the result will be a _____ integer.

When 2 integers with **different signs** are divided the result will be a _____integer.

Short Stack, Please!

Institute Notes

Concept: Investigate proportional and non-proportional relationships by measuring the heights of stacked objects.

TEKS Focus: **6.3—**The student solves problems involving proportional relationships.
7.3—The student solves problems involving proportional relationships.
8.3—The student identifies proportional relationships in problem situations and solves problems.

Also:
Grade 6
4A,5,12A,13A
Grade 7
4B, 4C, 14A, 14B, 15A
Grade 8
3A, 4, 5A, 5B, 15A, 16A, 16B

Overview: This activity explores the relationship between the number of items stacked and the height of the stack. Different types of stacks are investigated: objects stacked one above the other (like pancakes) and objects stacked by nesting one inside the other (like paper cups). The heights of which stacks represent proportional relationships and why? The characteristics of a proportional relationship explored in Day 1 *(Perplexing Puzzle* and *Trains)* will be used to make the distinction.

Materials:
Pattern Blocks (yellow hexagons)
Styrofoam cups of one size
Rulers with customary and metric scales
6" paper plates
Graph paper (1" or cm grid)
Markers or peel-and-stick dots
Graphing calculators

Procedure:

1. Participants are to work in groups of 2 to 4 with a set of pattern blocks (yellow hexagons), a ruler with a customary scale (inches) , markers, and graph paper.
2. Before displaying Activity 1 on the overhead, describe the following procedure for collecting the data: Stack one group of 4 hexagon-shaped pattern blocks (one on top of the other) and record the height of the stack (to the nearest half-inch). Then stack a second group of 4 pattern blocks on top of the first group, and record the height of the stack, etc. Ask participants to determine the dependent and independent variables in the data by having them answer the following question: Do we want to use the number of groups of hexagons to determine the height of the stack, or do

Math Notes:
Participants should be familiar with these terms as they are used in science in the middle grades. However, some review should take place to help participants better understand the concept of dependent and independent variables. Real-world examples like "The amount of money I get paid for working at the grocery store is dependent upon the number of hours I work" can be used. In this example, *the number of hours I work* is the

we want to use the height of the stack to determine the number of groups of hexagons? Since we are controlling the number of groups of hexagons that are used and then seeing what the result is for the height, we are using the number of groups (independent variable) to determine the height (dependent variable). In other words, the height of the stack in this experiment is dependent upon the number of groups of hexagons.

3. Have participants stack the groups of pattern blocks one at a time, measuring the height of the stack to the nearest half inch as they build it. Results are to be recorded on Activity Sheet 1 and graphed on 1" graph paper.
4. Have participants display their graphs around the room so that they can make comparisons and discuss the questions in Activity 1 in a debriefing session.
5. Record the results from the discussion of the questions in Activity 1 on Transparency 1 to further discuss the graphic representation of the data.
6. Before displaying Activity 2, explain the data gathering procedure and ask participants to identify the dependent and independent variables in this problem. (*The height of the stack of Styrofoam cups is dependent upon the number of cups stacked.*)
7. Participants will use Styrofoam cups, rulers, and graphing materials to do this activity. They are to place one cup with the rim on a flat surface and measure the height in cm, recording this measurement on the table in Activity 2. Participants will continue to stack one cup at a time on top of the other in a nesting manner measuring the height of the stack with two cups, three cups, etc. All measurements are to be recorded on the table in Activity 2.
8. When all data has been collected, participants are to graph their results and display them in the room.
9. In a debriefing session, use the questions on Transparency 2 to discuss the graphic representation of the data.

independent variable and *the amount of money I get paid* is the dependent variable. Have participants generate a number of these statements and identify the dependent and independent expressions from the statements.

It would be appropriate to use a "Gallery Tour" to enable participants to "get-up-and-move." As they walk around the room, they will write down how the graphs are alike and different. When the tour is completed, a debriefing should take place with the presenter facilitating the discussion.

In the debriefing session, ask participants to discuss other ways one could determine if this situation of nested cups represents a proportional or non-proportional relationship. Remind participants that articulation by the students is essential to their development of an understanding of proportional relationships and their characteristics.

Short Stack, Please!

Extensions: Ask participants to give examples of other objects that could be stacked to show a proportional relationship (*books and pancakes*); a non-proportional relationship (*things that "nest" such as cereal bowls and chairs*)

Have participants repeat this activity using a Styrofoam cup, a paper plate, a Styrofoam cup, a paper plate, etc. measuring the height of the stack after the addition of each cup and plate combination. Ask them to describe the type of relationship they have just modeled by this kind of stacking.

Assessment: Describe a situation that involves a proportional relationship. Explain your reasoning. Describe a situation that involves a non-proportional relationship and support your position.

Notes:

Short Stack, Please!

Short Stack, Please!

Activity 1

Put your yellow hexagon-shaped pattern blocks in groups of four and stack them one group at a time, recording the height in inches in the table below.

Number of Groups of Hexagons	Process (height of stack in inches)	Height of Stack (in inches)
1	1.5 (1)	1.5
2	1.5 (2)	3.0
3	1.5 (3)	4.5
4	1.5 (4)	6.0
5	1.5 (5)	7.5
6	1.5 (6)	9.0
7	1.5 (7)	10.5
10	1.5 (10)	15.0
20	1.5 (20)	30.0
25	1.5 (25)	37.5
g	1.5 (*g*)	*h*

a) How can you use your table to determine if this situation is a proportional relationship?
b) How can you use the graph to determine if this situation is a proportional relationship?
c) Write an equation for finding the height of any number of stacks, *g*.
d) How can you use the equation to recognize a proportional relationship? Explain.
e) Determine the height of 1 pattern block without measuring. Explain how you did this.
f) Write an equation for finding the height of any number, *n*, of pattern blocks without measuring.
g) What is the height of 50 blocks? 100 blocks? What equation did you use? Why?

TEXTEAMS Rethinking Middle School Mathematics: Proportionality Activity-5

Answers and Math Notes:

a. Compare the ratio of Height of Stack to Number of Groups of Hexagons for each of the rows in the table. What do you observe?

These ratios are all equivalent to 1.5 inches per group of hexagons.

1.5 is called a constant ratio and can be determined from the table of values. Explain what this ratio means in words.

This ratio represents 1.5 inches per group of hexagons in height. It also means that for every group of hexagons, there will be 1.5 inches in height.

b. The graph of a proporitonal realtionship is a straight line through the origin.

c. Participants should observe a pattern from their table in Activity 1. It is very important to discuss the process column of the activity as this leads to the generalization that can be expressed as the equation h=1.5g.

d. The equation is of the form $y = kx$. From this generalization, participants should be able to find the height of any number of groups of four hexagons in a stack by multiplying the number of groups in the stack by the constant of proportionality, 1.5.

For questions e, f, and g, participants must use the proportional relationship of the number of groups of hexagons to the height of the stack to determine the height of one block without measuring. Have the participants share their procedures for doing this, for there may be several strategies. For example, participants may reason that if there are 8 hexagons in 3 inches, each hexagon must be $\frac{3}{8}$ inches high;

or participants may express the relationship in a ratio $\frac{1.5 \text{ inches}}{4 \text{ hexagons}}$ or $\frac{3 \text{ inches}}{8 \text{ hexagons}}$ and then use division to express that ratio in an equivalent form of $\frac{0.375 \text{ inches}}{1 \text{ hexagon}}$;

participants may use a halving strategy in which they reason that 1.5 inches per 4 hexagons = 0.75 inches per 2 hexagons = 0.375 inches per 1 hexagon; or participants may use their graphic representation to determine that when g is $\frac{1}{4}$ group (or 1 hexagon) then h is $\frac{3}{8}$ = 0.375 inch.

In each of these strategies, the result is that h = 0.375n, where h is the height of the stack, n is the number of hexagons, and 0.375 is a new constant of proportionality describing the relationship of each hexagon to the height of the stack.

In the equations, $h = 1.5g$ and $h = 0.375n$, 1.5 and 0.375 are called the constants of proportionality. Writing an equation helps to symbolically represent a proportional relationship. We can say that the height of the stack is proportional to the number of hexagons in the stack.

By using the equation $h = 0.375n$, we can find that 50 hexagons are 18.75" high and 100 should be double that or 37.5" high. Compare using this generalized equation vs. cross-products which can have a very distorted meaning for students especially when they use the units and cross-multiply to get hexagon inches.

Short Stack, Please!

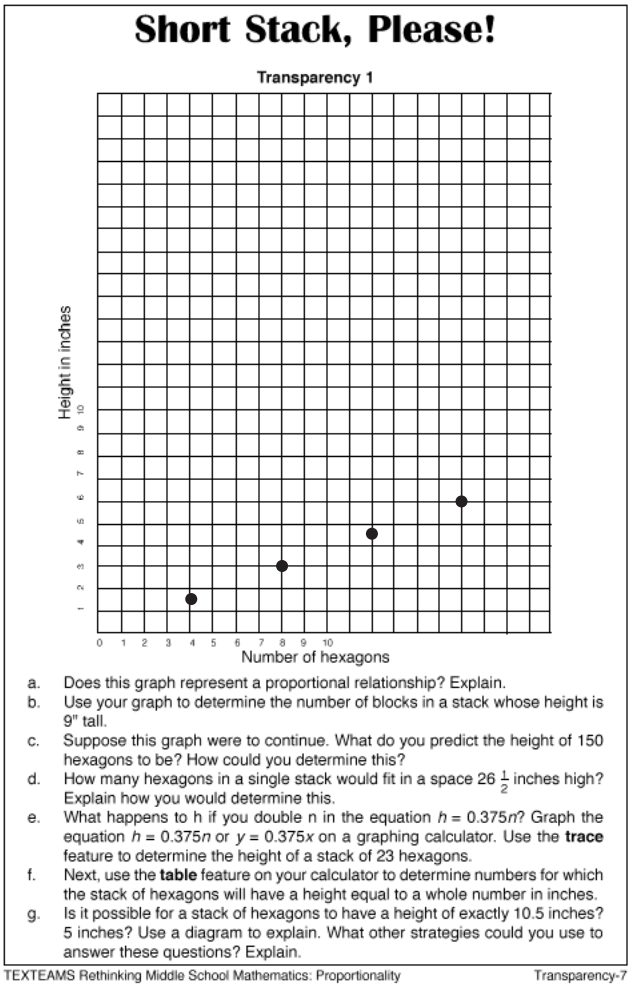
Activity 1

Put your yellow hexagon-shaped pattern blocks in groups of four and stack them one group at a time, recording the height to the nearest half-inch in the table below.

Number of Groups of Hexagons	**Process** (height of stack in inches)	**Height of Stack** (in inches)
1		
2		
3		
4		
5		
6		
7		
10		
20		
25		
g		*h*

a. How can you use your table to determine if this situation is a proportional relationship?
b. How can you use the graph to determine if this situation is a proportional relationship?
c. Write an equation for finding the height of any number of stacks, *g.*
d. How can you use the equation to recognize a proportional relationship? Explain.
e. Determine the height of 1 pattern block without measuring. Explain how you did this.
f. Write an equation for finding the height of any number, *n*, of pattern blocks without measuring.
g. What is the height of 50 blocks? 100 blocks? What equation did you use? Why?

Short Stack, Please!



Short Stack, Please!

Transparency 1

Height in inches

Number of hexagons

a. Does this graph represent a proportional relationship? Explain.
b. Use your graph to determine the number of blocks in a stack whose height is 9" tall.
c. Suppose this graph were to continue. What do you predict the height of 150 hexagons to be? How could you determine this?
d. How many hexagons in a single stack would fit in a space 26 $\frac{1}{2}$ inches high? Explain how you would determine this.
e. What happens to h if you double n in the equation $h = 0.375n$? Graph the equation $h = 0.375n$ or $y = 0.375x$ on a graphing calculator. Use the **trace** feature to determine the height of a stack of 23 hexagons.
f. Next, use the **table** feature on your calculator to determine numbers for which the stack of hexagons will have a height equal to a whole number in inches.
g. Is it possible for a stack of hexagons to have a height of exactly 10.5 inches? 5 inches? Use a diagram to explain. What other strategies could you use to answer these questions? Explain.

TEXTEAMS Rethinking Middle School Mathematics: Proportionality Transparency-7

Math Notes:
In this graph, the x-axis is labeled *Number of hexagons*. Guide participants to notice that in their data, $g = 1$ group of 4 hexagons. Therefore, you might want to put the point for 1 group on the 4th mark from 0 on the *x*-axis, etc.

Answers:

a. This graph represents a proportional relationship because the points lie on a straight line that passes through the origin.

b. A stack whose height is 9" tall contains 24 hexagons.

c. I would estimate about 54" tall for the height of 150 hexagons using the information from the previous problem. If 24 hexagons measure 9" in height and there are approximately 6 groups of 24 hexagons in 150, then I can multiply 9" by 6 to get about 54". By multiplying 150 by the actual height of 0.375", I get 56.25". Ask if anyone solved this using a different strategy.

d. Again, I could use the knowledge of **a** above: 24 hexagons are 9" tall. There are about 3 groups of 9" in 26 1/2". Therefore, 3 times 24 hexagons is 72 hexagons. This number will be slightly higher than the actual number. Next, I could multiply 0.375 times 71 to get 26.625 and then multiply 0.375 times 70 to get 26.25. Or, I could use the table feature of a graphing calculator and scroll to determine this. Since I only have 26 1/2" to accommodate a stack of hexagons, the stack can contain at most 70 hexagons. Ask if anyone used different reasoning to determine this solution.

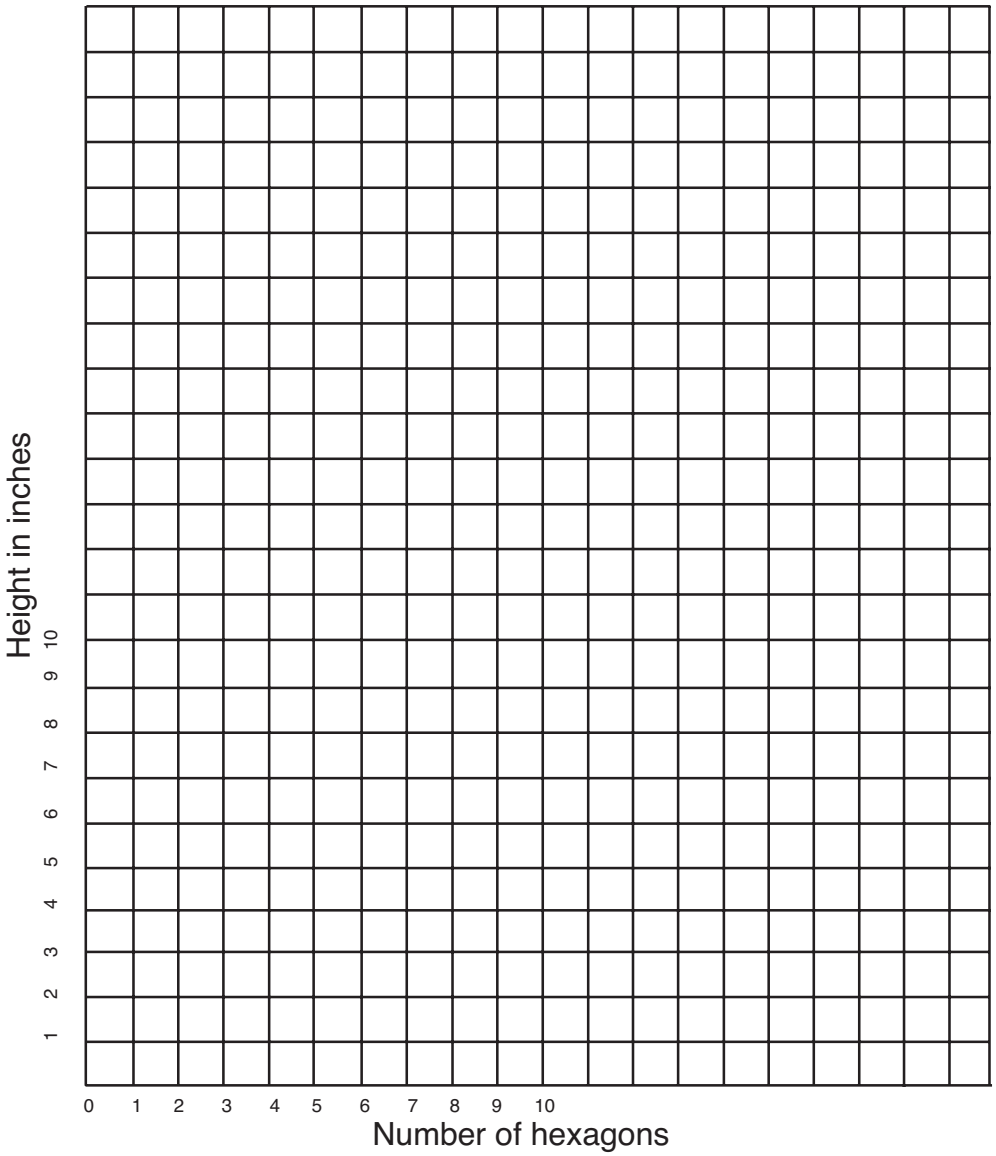
e. *h* is also doubled. A stack of 23 hexagons will be 8.625 inches in height.

f. The stack will have a height equal to a whole number of inches for 8, 16, 24, 32, 40,…or multiples of 8 because 0.375=3/8 and 3/8 times any multiple of 8 will give a whole number.

g. Yes, it is possible for a stack of 28 hexagons to have a height of exactly 10.5", but it is not possible to have a height of exactly 5". Using the table on a graphing calculator, one can scroll to find that 13 hexagons has a height of 4.875" and 14 hexagons has a height of 5.25".

Short Stack, Please!

Transparency 1



a. Does this graph represent a proportional relationship? Explain.

b. Use your graph to determine the number of blocks in a stack whose height is 9" tall.

c. Suppose this graph were to continue. What do you predict the height of 150 hexagons to be? How could you determine this?

d. How many hexagons in a single stack would fit in a space 26 $\frac{1}{2}$ inches high? Explain how you would determine this.

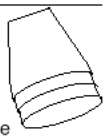
e. What happens to h if you double n in the equation $h = 0.375n$? Graph the equation $h = 0.375n$ or $y = 0.375x$ on a graphing calculator. Use the **trace** feature to determine the height of a stack of 23 hexagons.

f. Next, use the **table** feature on your calculator to determine numbers for which the stack of hexagons will have a height equal to a whole number in inches.

g. Is it possible for a stack of hexagons to have a height of exactly 10.5 inches? 5 inches? Use a diagram to explain. What other strategies could you use to answer these questions? Explain.

Short Stack, Please!

Short Stack, Please!



Activity 2

Place one Styrofoam cup with open end down on a flat surface and measure the height of the cup in cm. Place a second cup on top so that the cups are nested and measure the height of the stack. Continue stacking cups one at a time taking measurements to the nearest cm after each cup is added to the stack. Use the table below to record your results.

Number of Cups	**Process** (Height in cm)	**Height in cm**
1		
2		
3		
4		
5		
6		
10		
37		
n		*h*

a. What do you predict the height of 50 cups to be in cm? Explain your reasoning.

b. How could you determine the height of 100 cups? n cups?

c. Write an equation for finding the height of *n* cups.

d. Explain the connection between the cups stacked and your equation.

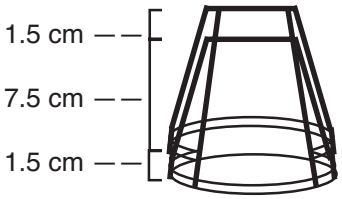
TEXTEAMS Rethinking Middle School Mathematics: Proportionality Activity-9

Answers:

a. and **b.**
Answers will vary according to size of cups.

c. The equation will be of the form $h = an + b$.

d. Have participants cut out a cross section of a cup that is stacked on top of another cup.

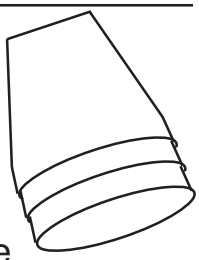


(8.5 oz. styrofoam cups)

Ask them to explain the "a" and "b" in their equation as represented in their model.

For example:
In the model alone, the equation is $y = 1.5x + 7.5$.
1.5 cm represents the space between the top of the bottom cup and the cup stacked on top. It is also the distance between the table and rim of the cup on top. The constant 7.5 is the distance between the rim of the cup on top and the top of the cup on bottom as illustrated. This distance stays the same no matter how many cups are stacked.

Short Stack, Please!



Activity 2

Place one Styrofoam cup with open end down on a flat surface and measure the height of the cup in cm. Place a second cup on top so that the cups are nested and measure the height of the stack. Continue stacking cups one at a time taking measurements to the nearest cm after each cup is added to the stack. Use the table below to record your results.

Number of Cups	**Process** (Height in cm)	**Height in cm**
1		
2		
3		
4		
5		
6		
10		
37		
n		*h*

a. What do you predict the height of 50 cups to be in cm? Explain your reasoning.

b. How could you determine the height of 100 cups? n cups?

c. Write an equation for finding the height of *n* cups.

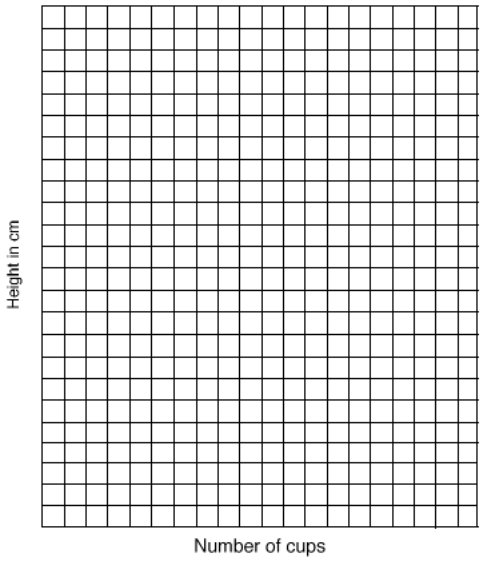
d. Explain the connection between the cups stacked and your equation.

Short Stack, Please!

Short Stack, Please!

Transparency 2

Use the data collected from Activity 2 to make a graph and answer the questions below.



a) How does this graph differ from the graph for Activity 1?
b) Does this graph represent a proportional or non-proportional relationship? Explain.
c) Enter the equation for this relationship in a graphing calculator and use **tbl set** to set the initial value for x at 0 and △**tbl** at 1. Leave the other settings on **auto.**
d) Next key in **2nd table** to view a table of values for this function. Use the arrow keys to scroll the table to answer the following questions: What would be the height of 150 cups? Is this twice the height of 75 cups? Explain. If you know the height of 30 cups, can you predict the height of 120 cups? Why or why not?

Reason and Communicate:
It is important to point out that, while this graph does not represent a proportional relationship, there is proportionality represented by the slope of the line where

$\frac{\Delta y}{\Delta x}$ is a constant ratio

for every pair of points on the line.

Ask participants to explain the meaning of an ordered pair (x,y) on the graph in words. (They should explain that x cups is y cm in height.)

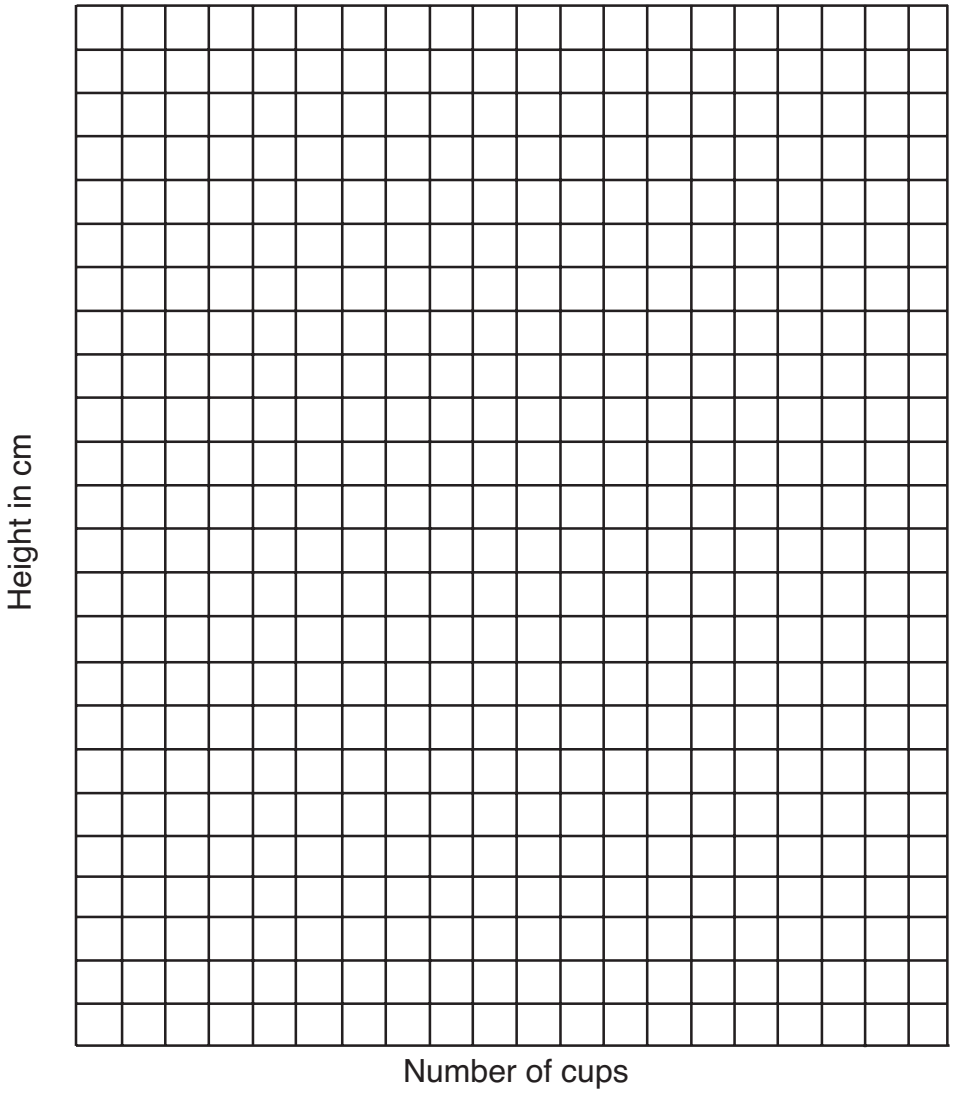
Ask them if there is a way to stack these cups differently so that a proportional relationship would be represented? Have them explore with their conjectures and explain their results using multiple representations (table, graph, diagram, equation, verbal explanation).

Answers:
a. The graph of this line does not pass through the origin.
b. This graph represents a non-proportional relationship since it is the graph of an equation that is not of the form $y = kx$.
c. Answers will vary according to the size of the Styrofoam cup used.
d. Answers will vary.

Short Stack, Please!

Transparency 2

Use the data collected from Activity 2 to make a graph and answer the questions below.



a. How does this graph differ from the graph for Activity 1?
b. Does this graph represent a proportional or non-proportional relationship? Explain.
c. Enter the equation for this relationship in a graphing calculator and use **tbl set** to set the initial value for *x* at 0 and △ **tbl** at 1. Leave the other settings on **auto.**
d. Next key in **2nd table** to view a table of values for this function. Use the arrow keys to scroll the table to answer the following questions: What would be the height of 150 cups? Is this twice the height of 75 cups? Explain. If you know the height of 30 cups, can you predict the height of 120 cups? Why or why not?

Questioning and the 5E Lesson

Subject/Lesson: *Grade 6 Mathematics: Modeling Subtraction of Fractions with Regrouping* Objective: *6.2A, 6.2B, 6.2D, 6.11A, 6.12A*

Stage of 5E Lesson	Question	Bloom's Level
Engage *Gets the students' minds focused on the topic.*	**Missing Parts** • How can you use the given information to create a model? *(create)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create
Explore-cycle 1 **(model subtraction of fractions with regrouping and LIKE denominators)** *Provides students with a common experience.*	**How Much Remains?** • What role does estimation play in solving these types of problems? *(understand)* • Which color rod should represent 1 whole? *(evaluate)* Why? *(analyze)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create
Explain-cycle 1 *Teaches the concept with interaction between the teacher and students.*	**Debrief: How Much Remains?** **Optional: Take Me Away!** • How can you tell if an "exchange" of rods is necessary? *(analyze)* • How are these subtraction problems similar to or different from those done previously (without regrouping)? *(analyze)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create

Explore-cycle 2 **(model subtraction of fractions with regrouping and UNLIKE denominators)** *Provides students with a common experience.*	**"P" is for Parts of Wholes** • How are these problems similar to or different from those in **How Much Remains?** *(analyze)* • When have you solved a problem like this before? *(remember)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create
Explain-cycle 2 *Teaches the concept with interaction between the teacher and students.*	**Debrief: "P" is for Parts of Wholes** • How are the rules for subtracting mixed numbers similar to or different from subtracting proper fractions? *(analyze)* • How would you explain the process for solving a problem like this without using a model? *(apply)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create
Elaborate *Provides opportunity for students to apply the concept in a new situation.*	**Independent Practice** • How do you know if your answer is reasonable? *(analyze)* • How could you verify your answer? *(understand)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create
Evaluate *Allows students to demonstrate understanding of the concept and facts.*	**Corn and Sweet Peas + 4 selected response items** • What context clues in the problem could you use to help decide which operation is necessary? *(remember)* • How can you tell if the numbers are in a correct format before attempting a calculation? *(understand)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create

Missing Parts

Use Cuisenaire® Rods to find the missing number and record your observations.

Equivalent Fractions	Equivalent Fraction Models	Observations
$2\frac{1}{3} = \square\frac{4}{3}$		
$\square\frac{1}{2} = 3\frac{3}{2}$		
$2\frac{2}{5} = 1\frac{\square}{5}$		
$3\frac{1}{\square} = 2\frac{4}{\square}$		

How much remains?

Use Cuisenaire® Rods to model each situation and to complete the table.

Situation	Model	Procedure	What did you exchange?
Sugar Mrs. Ibarra had 2 cups of sugar in her pantry. Her neighbor borrowed $\frac{3}{4}$ cup of sugar. How much of the sugar remains in Mrs. Ibarra's pantry?			
Music A certain CD will hold $1\frac{1}{6}$ hours of music. Michael has already recorded $\frac{5}{6}$ hours of music on the CD. What part of an hour remains unused?			
Ribbon Rebekah had $2\frac{1}{3}$ yards of ribbon. She used $1\frac{2}{3}$ yards of ribbon to trim the bottom of a skirt. How much ribbon remains?			

How much remains?

Situation	Model	Procedure	What did you exchange?
Lawns Jesse earns extra money by mowing lawns for 5 of his neighbors. On Saturday, he had mowed $3\frac{4}{5}$ lawns before it started to rain. How many lawns remain for Jesse to mow?			
Pizza Ameena had $3\frac{3}{8}$ pizzas left after a party. Her brother, Musa, ate $1\frac{5}{8}$ pizzas when he got home from soccer practice. How much pizza remains?			

Take Me Away!

Simplify each subtraction problem.

1. $5 - 3\frac{1}{5}$

2. $3\frac{1}{4} - \frac{3}{4}$

3. A recipe calls for 3 cups of flour. Mrs. Shipley only has $2\frac{1}{3}$ cups of flour. How much more flour does she need?

4. $4\frac{2}{7} - 1\frac{5}{7}$

5. $2 - \frac{9}{10}$

"P" is for Parts of Wholes

Model each problem and record your observations.

Word Problem	Model	Procedure	Observations
Pounds of Pasta Bae's family ate $3\frac{1}{2}$ pounds of pasta at Pastaria. Barid's family ate $2\frac{3}{4}$ pounds of pasta at the same restaurant. How much more pasta did Bae's family eat?			
Personal Pan Pizza Cade bought 3 cases of frozen personal pan pizzas. He sold $1\frac{2}{5}$ cases. How many cases of frozen personal pan pizzas were left?			
Popsicles Easter ordered $2\frac{1}{5}$ cases of popsicles for the party. All but $\frac{2}{5}$ of the cases were eaten. How many cases of popsicles were eaten?			

"P" is for Parts of Wholes

Word Problem	Model	Procedure	Observations
Pita Pockets Dacian's Bakery baked $4\frac{1}{3}$ pounds of pita pockets. Dakarai's Bakery baked $1\frac{3}{4}$ pounds of pita pockets. How much more pita pockets did Dacian's Bakery bake than Dakarai's Bakery?			
Popcorn Abagail and three of her friends bought popcorn from the school fundraiser. Abagail and Abby ate $2\frac{3}{4}$ bags of the popcorn during the basketball game. Abella and Aaralyn ate $1\frac{7}{8}$ bags of the popcorn during lunch. How much more popcorn did Abagail and Abby eat than Abella and Aaralyn?			

Independent Practice

Modeling fraction subtraction that requires borrowing:

$3\frac{1}{3}$ unlike denominators $3\frac{4}{12}$ $\frac{9}{12}$ is to large to subtract from $\frac{4}{12}$ $2\frac{16}{12}$

$-1\frac{3}{4}$ $-1\frac{9}{12}$ $-1\frac{9}{12}$

$1\frac{7}{12}$

Since, there are unlike denominator a common denominator must be found first. The LCM of 2 and 4 is 12, so the common denominator would be 12.

$3\frac{4}{12}-1\frac{9}{12}$

3 decreased by 1.
$3-1=2$

$3\frac{4}{12}$ $2\frac{16}{12}$

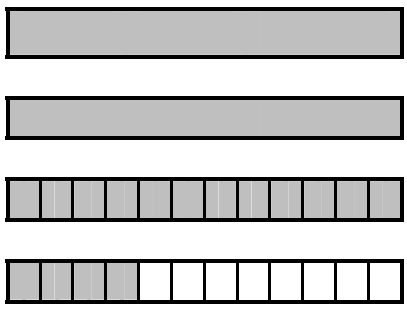
$\frac{4}{12}$ increased by 1.

$\frac{4}{12}+1$

$\frac{4}{12}+\frac{12}{12}=\frac{16}{12}$

Model of $\mathbf{3\frac{1}{3}}$.

Model of $\mathbf{2\frac{16}{12}}$.



Model of $\mathbf{2\frac{16}{12}}$ less $\mathbf{1\frac{9}{12}}$.

Use a model to solve the problem.

1. $4-1\frac{1}{5}$

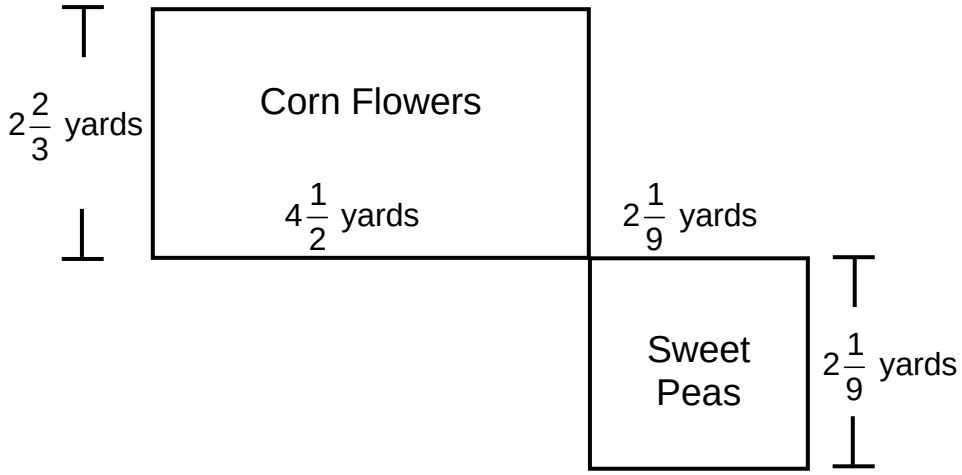
Complete the table.

Fraction Problem	Model	Process
2. $7\frac{1}{2}-2\frac{3}{4}$		
3. Fadey had $2\frac{1}{6}$ ounces of salt water solution. He used $\frac{3}{4}$ ounce of the solution for lab research. How many ounces of salt water solution does Fadey have left?		
4. $4\frac{1}{3}-3\frac{3}{5}$		
5. Gainella bought 4 yards of ribbon. She used all but $2\frac{3}{7}$ yards of the ribbon. How much of the ribbon did Gainella use?		

Corn and Sweet Peas

Habib needs to fence in his corn flower and sweet pea gardens shown below. How much more fencing material will he need to purchase for the corn flower garden than the sweet pea garden? Justify your answer.

Habib Garden



FOR TEACHER USE ONLY:

a. YES NO Student arrives at a correct solution?

	4	3	2	1
b. Conceptual Knowledge				
c. Procedural Knowledge				
d. Communication				

1 Kacy completed $1\frac{1}{3}$ of the animation slides on Monday and $2\frac{1}{4}$ of the slides on Wednesday. Lacy completed $5\frac{1}{6}$ of the slides on Friday. How many more slides did Lacy complete than Kacy?

A $1\frac{7}{12}$

B $2\frac{7}{12}$

C $3\frac{7}{12}$

D $9\frac{3}{4}$

2 Ida made 3 cases of homemade grape jelly. She sold all but $1\frac{1}{6}$ cases. How many cases of grape did Ida sell?

A $1\frac{5}{6}$

B $2\frac{5}{6}$

C $3\frac{1}{6}$

D $4\frac{1}{6}$

3 Jackson has $5\frac{1}{2}$ gallons of red paint. He used $1\frac{3}{4}$ gallons to paint the barn door. How much paint does Jackson have left?

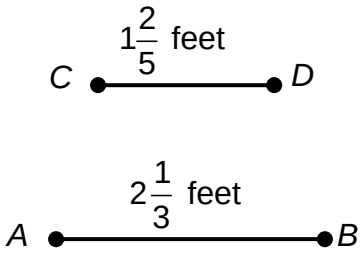
A $3\frac{1}{4}$

B $3\frac{3}{4}$

C $4\frac{3}{4}$

D $7\frac{1}{4}$

4 Line segment *AB* and line segment *CD* are shown below.



How much longer is line segment *AB* than line segment *CD*?

A $3\frac{11}{15}$

B $2\frac{11}{15}$

C $1\frac{14}{15}$

D $\frac{14}{15}$

Questioning and the 5E Lesson

Subject/Lesson: *Grade 7 Mathematics: Modeling Subtraction of Integers*

Objective: *7.2C, 7.15A, 7.15B*

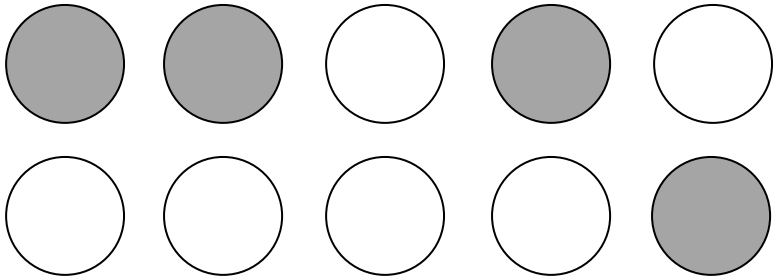
Stage of 5E Lesson	Question	Bloom's Level
Engage *Gets the students' minds focused on the topic.*	**How Many Counters** • What does a shaded/unshaded circle represent? *(remember)* • How could the value of Alejandro's collection be represented in a different way? *(understand)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create
Explore *Provides students with a common experience.*	**Complete the tables using 2-color counters & number line models** • How can you tell if the collection of counters needs to be modified prior to removing any counters? *(analyze)* • How is subtraction represented on a number line? *(understand)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create

Explain *Teaches the concept with interaction between the teacher and students.*	**Debrief: Explore** • How does subtraction with integers compare to subtraction with whole numbers? *(analyze)* • What "rules" could we develop for subtracting integers? *(create)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create
Elaborate *Provides opportunity for students to apply the concept in a new situation.*	**Independent Practice** • How do you know if your answer is reasonable? *(analyze)* • How could you verify your answer? *(understand)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create
Evaluate *Allows students to demonstrate understanding of the concept and facts.*	**Mystery Integers + 4 selected response items** • How could a model help you determine the solution? *(apply)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create

Key	
Positive	Negative

How Many Counters?

From a bag containing positive and negative counters, Alejandro drew the collection of counters shown below.



Problem 1

Alejandro's friend Alice removed 3 counters from Alejandro's collection. After the 3 counters were removed, the value of the collection was 1. Which counters did she remove? Justify your answer.

Problem 2

Alice wants to remove 6 negative counters from Alejandro's collection. How can this be possible?

Complete the table.

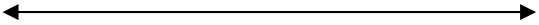
- Select the appropriate number of positive or negative counters.
- Remove the specified counters. (Hint: You may need to add neutral pairs.)
- Draw a picture of your collection of counters.
- Determine the value of the collection.
- Write a number sentence and a verbal description.

Number of Counters to Draw	Remove	Picture Two-Color Counters	Value	Number Sentence	Verbal Description
8 negative counters	3 negative counters				I removed three negative counters from eight negative counters and I am left with five negative counters.
6 positive counters	2 positive counters				
5 negative counters	1 positive counter				
1 positive counter	3 negative counters				
7 negative counters	2 positive counters				
6 positive counters	3 negative counters				

1. What patterns do you notice in the relationship between the number sentences and their values?

Complete the table.

- Draw a picture to model the problem using two-color counters.
- Draw a picture to model the problem using a number line.

Problem	Picture Two-Color Counters	Picture Number Line	Solution
6 − 3			
−3 – (−1)			
−3 − 2			
4 – (−3)			
−5 – 2			

Independent Practice

Remember that it may be necessary to add neutral pairs before the required number of tiles can be removed.

A **Neutral Pair** contains one positive and one negative. (+ –)

Example 1: –2 – 3 =

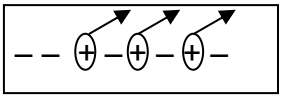
Step 1: Think: Start with 2 negative tiles then remove 3 positive tiles.

Step 2: Draw a picture. – –

Step 3: There are no positive tiles to remove, so add 3 neutral pairs.

– – + – + – + –

Step 4: Remove the 3 positive tiles.



Step 5: What remains is the value, so

–2 – 3 = –5

Example 2: –4 – (–5) =

Step 1: Think: Start with 4 negative tiles then remove 5 negative tiles.

Step 2: Draw a picture. – – – – –

Step 3: There are only 4 negative tiles to remove, so add 1 neutral pair.

– – – – – + –

Step 4: Remove the 5 negative tiles.

Step 5: After the tiles are removed, what remains is the value, so

–4 – (–5) = 1

Complete:

1. Explain how to subtract (remove) –4 from –2 and find the value of the expression.

2. Explain how to subtract (remove) 5 from –3 and find the value of the expression.

Solve:

3. 8 – 5 =

4. –6 – (–4) =

5. –12 – 23 =

6. 16 – (–3) =

7. At 6:00 A.M. on a cold winter morning in Buffalo, New York, the temperature was –22°F. At noon the temperature was 18°F. By how many degrees did the temperature change from 6:00 A.M. to noon?

8. On the first play of the game, the school football team was given a penalty of –15 yards. After reviewing the play, the referee removed 10 yards from the penalty. How many yards was the penalty after the referee removed the yards?

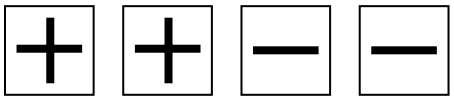
9. –19 – 12 =

10. 5 – (–39) =

11. 47 + –34 =

12. –8 – 6 =

13. Rusty drew the collection of tiles below from a bag of positive and negative tiles. He was asked to remove 6 positive tiles. What should be his first step?



Mystery Integers

Alopie and Farid were playing the new hit game, Mystery Integers. To help Alopie guess the mystery integer, Farid gave her the following clue.

"If I remove negative 8 from the mystery integer, the result will be positive 4."

What is the mystery integer? Justify your answer.

FOR TEACHER USE ONLY:

a. YES NO Student arrives at a correct solution?

	4	3	2	1
b. Conceptual Knowledge				
c. Procedural Knowledge				
d. Communication				

1 Which expression gives the same value as –7 – (–5)?

A –7 – 5

B 7 – (–5)

C –7 + 5

D 7 + 5

2 Ali drew 3 negative and 4 positive tiles from a bag. If he removes 2 positive tiles from his collection, what will be the value of his tiles?

A –2

B –1

C 1

D 3

3 Evaluate the expression below.

–3 + 4 – (–5) +7 – 6

A –3

B –1

C 7

D 8

4 Deloris drew the tiles shown. She was asked to subtract 3 negative tiles.

+ + – + +

Which diagram shows a correct first step for solving this problem?

A + + – + + + – +

B + + – + + – –

C + + – + + + – + –

D + + – + + + +

Questioning and the 5E Lesson

Subject/Lesson: *Grade 8 Mathematics: Proportional and Non-proportional Relationships*

Objective: *8.2D, 8.3A, 8.3B, 8.4A*

Stage of 5E Lesson	Question	Bloom's Level
Engage *Gets the students' minds focused on the topic.*	**Recall information about proportional relationships** **1. Students work with a partner for 3 minutes writing down everything they can remember about proportions and proportional relationships.** **2. Prompt groups to share some of their ideas with the class. Everyone can add/modify their lists during the sharing.** • What is a proportion? *(remember)* • When are proportions typically used? *(remember)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create
Explore *Provides students with a common experience.*	**What's the Difference?** • How are Patterns A and B similar? *(analyze)* • How are Patterns A and B different? *(analyze)* • What does the intersection of a Venn diagram represent? *(remember)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create

Explain *Teaches the concept with interaction between the teacher and students.*	**Debrief: Explore** • How can the data in a table be used to determine if the relationship is proportional or non-proportional? *(analyze)* • What kind of relationship is found between the *x* and *y* values in a proportional relationship? *(evaluate)* • How are predictions made from various representations of data? *(apply)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create
Elaborate *Provides opportunity for students to apply the concept in a new situation.*	**Independent Practice** • What are the characteristics of a proportional relationship? *(understand)* • What are the characteristics of a non-proportional relationship? *(understand)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create
Evaluate *Allows students to demonstrate understanding of the concept and facts.*	**Relaxing Getaways + 4 selected response items** • How do you know your answer is reasonable? *(analyze)* • How could you verify your answer? *(understand)*	○ Remember ○ Understand ○ Apply ○ Analyze ○ Evaluate ○ Create

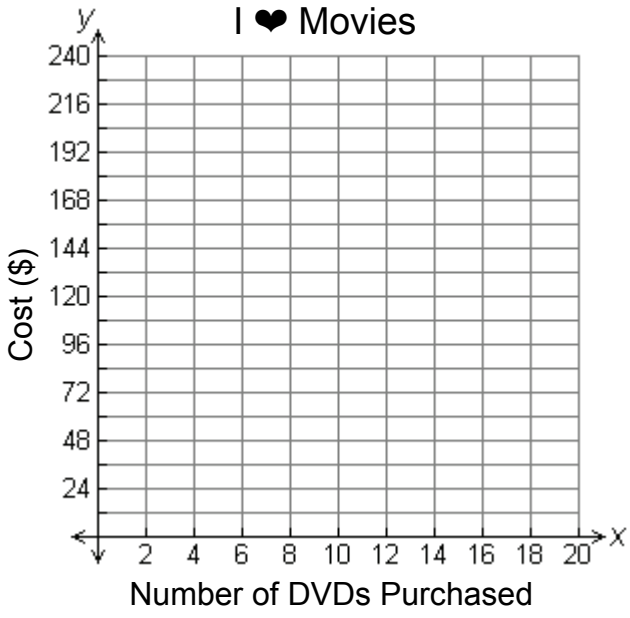
What's the Difference?

Pattern A:

Members of the "I ❤ Movies" club are able to purchase DVDs at a discounted rate of only $12 each. Complete the table and graph below to show the cost for members who purchase various numbers of DVDs:

I ❤ Movies

# of DVDs Purchased (x)	Cost (y)	Write the ratio $\frac{y}{x}$ as a:	
		Fraction	Decimal
2			
5			
11			
16			
x			



a) What patterns do you notice in the table and/or the graph?

b) What would be the cost to a member who purchases 15 DVDs? Justify your answer.

c) If a member only wants to spend $90, what is the maximum number of DVDs he can purchase? Justify your answer.

d) What is the cost to a member who doesn't purchase any DVDs? Where is this cost represented on the graph?

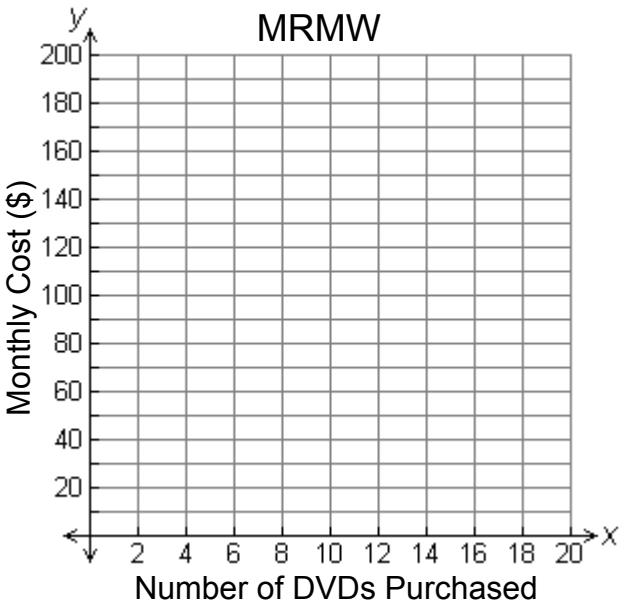
What's the Difference?

Pattern B:

The "Movies Rock My World" club (also known as MRMW) charges its members a monthly fee of $15. In return, members may purchase DVDs from the club's selection for only $8 each. Complete the table and graph below to show the cost for 1 month to members who purchase various numbers of DVDs:

MRMW

# of DVDs Purchased (x)	Monthly Cost (y)	Write the ratio $\frac{y}{x}$ as a:	
		Fraction	Decimal
5			
10			
15			
20			
x			



a) What patterns do you notice in the table and/or the graph?

b) What would be the cost to a member who purchased 12 DVDs in 1 month? Justify your answer.

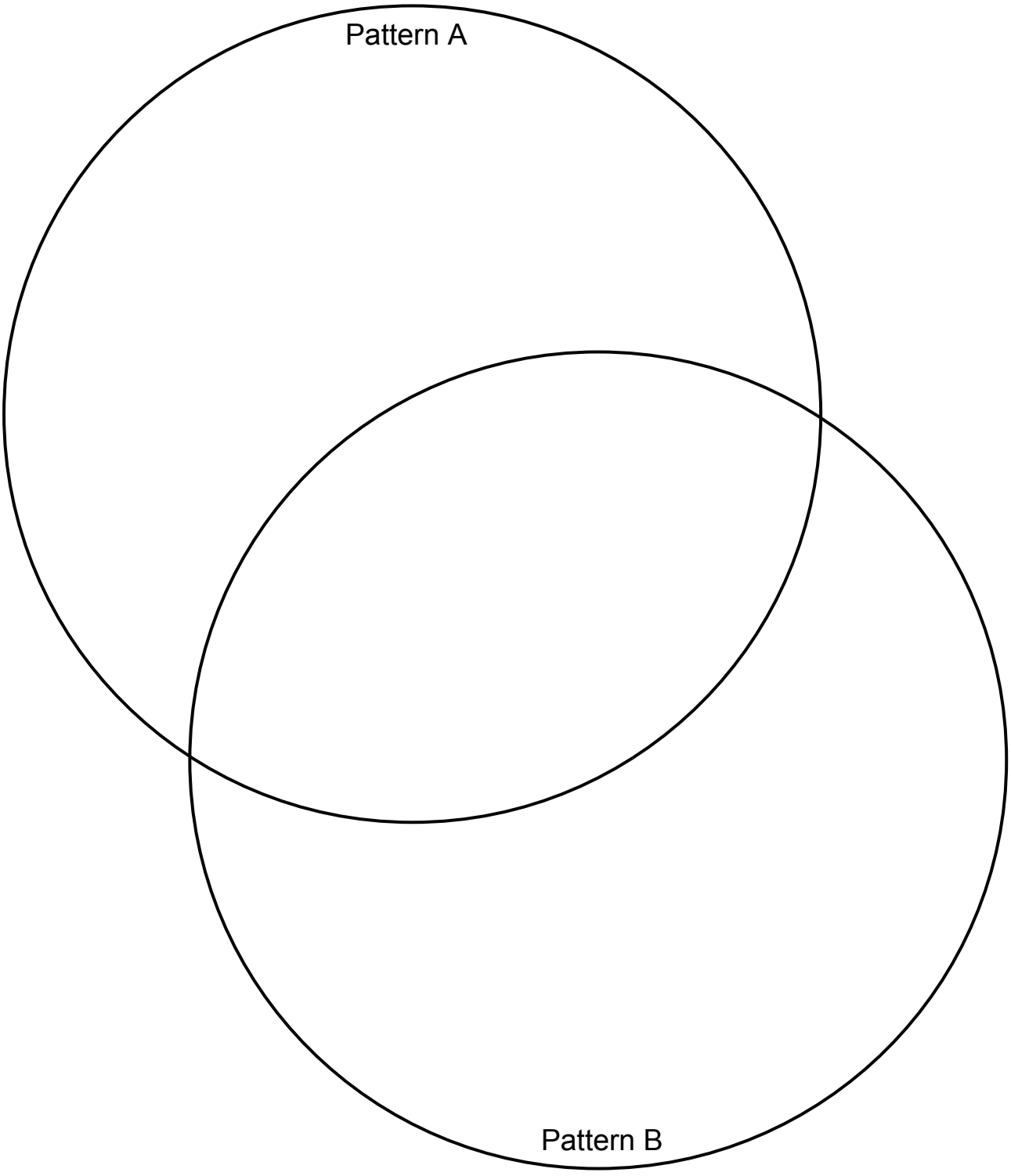
c) If a member only wants to spend $80, what is the maximum number of DVDs he can purchase that month? Justify your answer.

d) What is the cost to a member who doesn't purchase any DVDs during a month? Where is this cost represented on the graph?

What's the Difference?

Summary:

Use the Venn diagram below to compare and contrast Pattern A and Pattern B. Record your observations of each pattern in the appropriate region of the Venn diagram.



Independent Practice

A proportional relationship can be identified by looking at the:

a) table to see if there is a constant rate of change and a constant of proportionality
b) graph to see if it is a line that passes through the origin
c) equation to see if it fits the form $y = kx$

Proportional Relationship

A candy necklace costs $1.20. Jaiden wants to buy candy necklaces for all of her friends, but only has $15.00 to spend. How many necklaces can Jaiden purchase with $15?

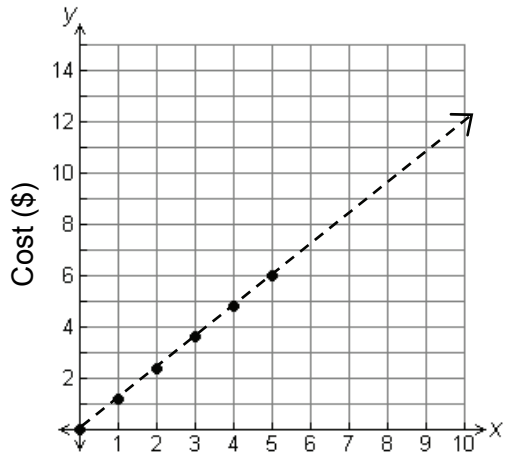
Jaiden's Candy Necklaces

# of necklaces (x)	Cost (y)	Value of $\frac{y}{x}$
1	$1.20	1.2
2	$2.40	1.2
3	$3.60	1.2
4	$4.80	1.2
5	$6.00	1.2
x	$1.2x$	

There is a constant rate of change of $1.20 per necklace and the equation fits the form $y = kx$

There is a constant of proportionality of 1.2

Jaiden's Candy Necklaces



Number of Necklaces Purchased

The graph is a line which passes through the origin.

To determine the number of candy necklaces Jaiden can purchase with $15, extend the table or the graph, or solve the equation $15 = 1.2x$. Jaiden can purchase 12 candy necklaces with $15.

Non-proportional Relationship

A health club charges $30 per month as a membership fee. Members can enroll in fitness classes for $10 per class. How many classes could Monica take in one month for $100?

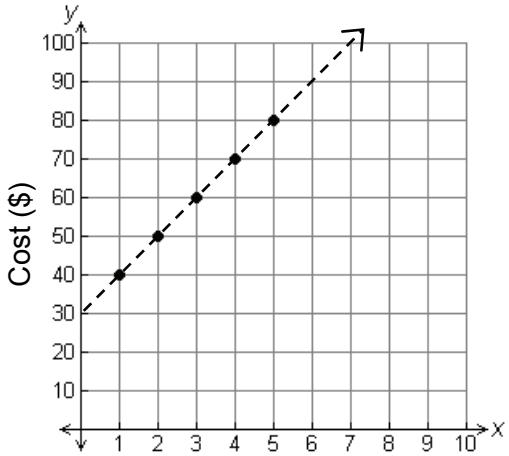
Monica's Fitness Classes

# of classes (x)	Cost (y)	Value of $\frac{y}{x}$
1	$40	40
2	$50	25
3	$60	20
4	$70	17.5
5	$80	16
x	$30 + 10x$	

There is a constant rate of change of $10 per class but the equation does NOT fit the form $y = kx$

There is NOT a constant of proportionality

Monica's Fitness Classes



Number of Classes Taken

The graph is a line but does NOT pass through the origin.

To determine the number of classes Monica can take with $100, extend the table or the graph, or solve the equation $100 = 30 + 10x$. Monica can take 7 classes for $100.

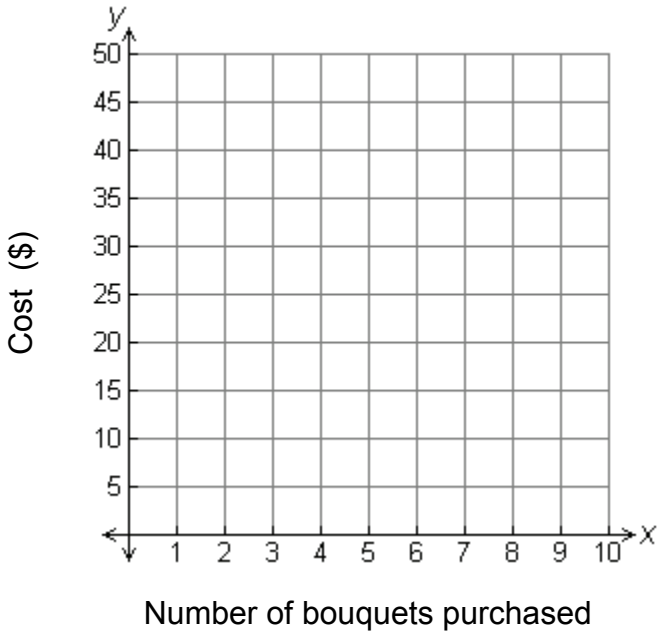
1 A candy necklace costs $1.20. Jaiden wants to buy candy necklaces for all of her friends, but only has $15.00 to spend. Jaiden asks her brother to drive her to the store so that she can purchase the candy necklaces for her friends. He agrees, but only after she gives him $5.00 for gas money. Now Jaiden only has $10.00 to spend. What is the maximum number of candy necklaces she can purchase? Justify your answer.

2 Tulips are on sale at Flowers R Us this week. You can buy a bouquet of a half-dozen tulips for $5.00. Complete the table and the graph based on this information:

Tulip Bouquets

Number of bouquets purchased	Cost in dollars
1	
2	
3	
4	
5	
10	
x	

Tulip Bouquets



a) Does this describe a proportional relationship? Justify your answer.

b) How much would it cost to purchase 25 bouquets? Justify your answer.

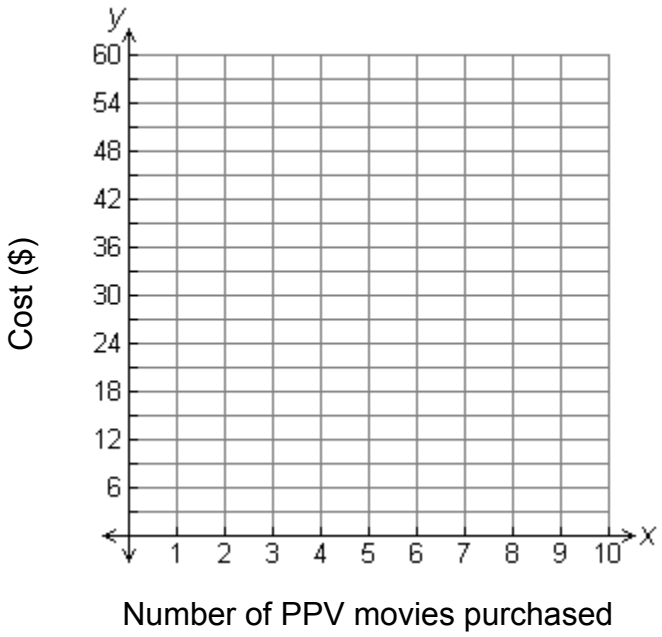
c) How many bouquets could be purchased with $45? Justify your answer.

3 A cable company charges $30 per month for basic service. Pay-per-view movies can be purchased for an additional $3.00 per movie. Complete the table and the graph based on this information:

Pay-per-view Movies

Number of PPV movies purchased	Cost in dollars
1	
2	
3	
4	
5	
10	
x	

Pay-per-view Movies



a) Does this describe a proportional relationship? Justify your answer.

b) How much would it cost to purchase 8 pay-per-view movies? Justify your answer.

c) How many pay-per-view movies could be purchased with $50? Justify your answer.

Relaxing Getaways

The Relaxing Getaways Resort charges $260 per night for 2 people. For each additional guest, there is a fee of $20 per person. Is this a proportional relationship? Justify your answer.

FOR TEACHER USE ONLY:

a. YES NO Student arrives at a correct solution?

	4	3	2	1
b. Conceptual Knowledge				
c. Procedural Knowledge				
d. Communication				

1 Pizza Plaza is having a special where medium pizzas cost $5 each. Toppings can be added for $0.75 each. Which equation could be used to determine the total cost, c, of 2 medium pizzas with 3 toppings each?

A $c = 0.75 \cdot 2 + 5$

B $c = 0.75 \cdot 3 + 5$

C $c = 2(0.75 \cdot 2 + 5)$

D $c = 2(0.75 \cdot 3 + 5)$

2 A car rental company charges $0.85 per mile traveled. Which proportion could NOT be used to find x, the cost of renting a car to travel 200 miles?

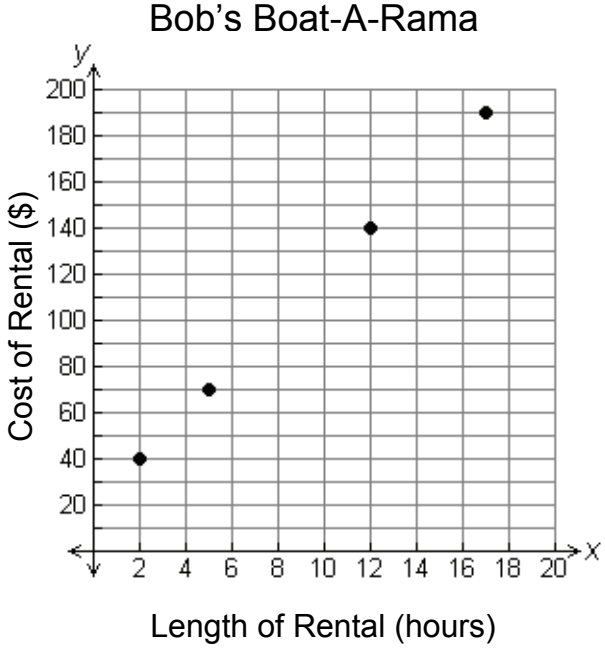
A $\frac{x}{0.85} = \frac{1}{200}$

B $\frac{1}{200} = \frac{0.85}{x}$

C $\frac{0.85}{1} = \frac{x}{200}$

D $\frac{200}{x} = \frac{1}{0.85}$

3 The graph below represents the cost of renting a boat from Bob's Boat-A-Rama during the summer season.



What would be the cost of renting a boat for 1 hour?

A $20

B $30

C $40

D $50

4 Mr. Meadows needs to ship some boxes and has compared prices from 4 different shipping companies. Which shipping company's price table is based on a constant unit price?

A

I Ship 4U

Weight of box	Cost to ship
10 lbs	$47
20 lbs	$102
30 lbs	$145
40 lbs	$212

C

Shippers Away

Weight of box	Cost to ship
10 lbs	$53
20 lbs	$105
30 lbs	$158
40 lbs	$210

B

Ships Ahoy

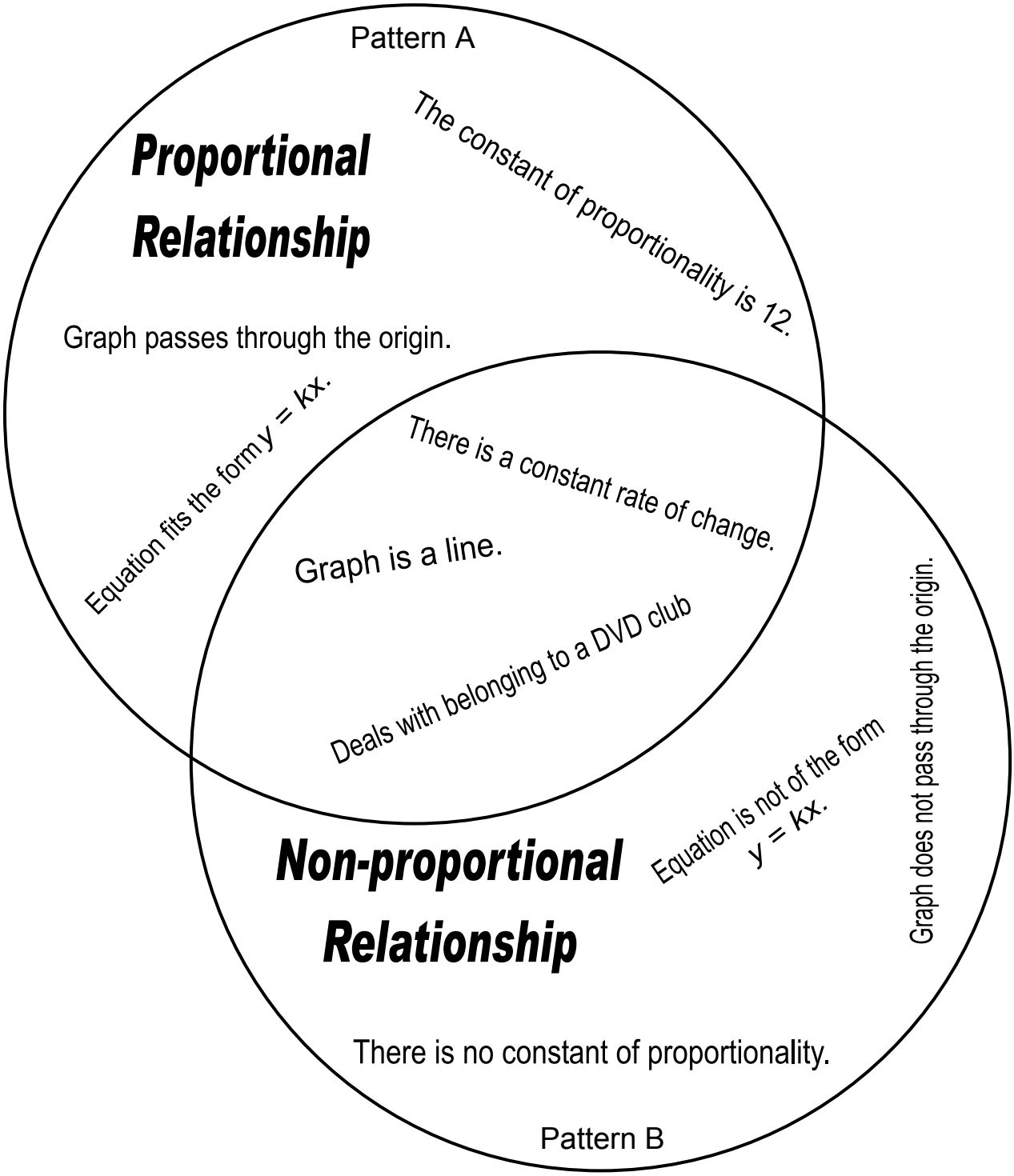
Weight of box	Cost to ship
10 lbs	$45
20 lbs	$95
30 lbs	$145
40 lbs	$195

D

Shipping Tales

Weight of box	Cost to ship
10 lbs	$56
20 lbs	$112
30 lbs	$168
40 lbs	$224

What's the Difference? Possible Responses:



My Implementation Plan

1. When do I plan to implement this lesson?

2. What do I need before I can implement this lesson? (Manipulatives? Copies? Technology? How many? Additional support from Wanetta? Abe? Sana?)

3. How and when will I get what I need? (Who can help me and how will I communicate my needs to them? Do I need to talk with a dept chair/campus specialist/district specialist to get manipulatives?)

Request for: Wanetta Abe Sana _____

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